Optimizing Response Time to Minimize Spreading of Conspiracy Theories in Dynamic Social Networks

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Abstract. The complexity and dynamicity of the social networks makes their analysis NP-hard problem. The operational methods and the traditional community detection methods fall short of modeling the complex activities in social networks. For such reasons, Systems Thinking and Modeling approaches to model and control the interactions between communities in both online and offline social networks was introduced. These methods implement useful multidisciplinary tools to interpret the actions of the users and dynamic activities of the communities in the evolutionary online (offline) social networks. In this paper, we utilized the organizational cybernetics approach to control/monitor the malicious information spread between communities, and the stochastic one-median problem to minimize the response time of the operation level to eliminate the spread across the online (offline) social networks. The performance of these methods was applied to a Twitter network related to an armed protest demonstration against COVID-19 lockdown at Michigan state in May 2020. The outcomes illustrate the dynamicity of the network, optimize the monitoring process of the operation level, minimize the response time to malicious information spread, and measure the performance of the system to respond to the second stochastic malicious action spread in the network.

Keywords: Systems Thinking, Complexity Theory, System Dynamics, Organizational Cybernetics, Stochastic One-median Problem, Misinformation, Conspiracy Theories, COVID-19.

1 Introduction

Complexities in monitoring, controlling, and analyzing the spread of so-called conspiracy theories, misinformation, disinformation, and fake news on social networks (in this research we call as malicious information) make it NP-hard problem. The tremendous growth in the number of users and communities in recent years with dynamic interactions among them makes analysis of such dynamic networks a complex problem. With the resources invested by big tech companies, government agencies, and academic institutions to identify the strategies to eliminate such abnormalities from online platforms, this problem is still at large. The large user base of the social networks, their dynamic interactions, and the impact of actions of the users in both online and offline [1] environments dictate the need for fast and automated monitoring strategies. And this demands early detection of these behaviors to limit the spread of malicious activity [2].

However, in response to such problems and complexities in the analysis, authors in [3] introduced the systems thinking methods to overcome these issues. Correspondingly, the authors stated that these multidisciplinary methods can supplement the managers with useful tools to simplify the monitoring process and reduce the time to respond to any malicious information spread in dynamic social networks [3]. In addition, the systems thinking methods can provide more details related to the evolution of the social network and help to relax the complexities of the dynamic networks analysis and implement optimized systematic solutions to reduce the network vulnerability from any extensive malicious information spread between communities in both online and offline environments.

In this paper, we present a systematic approach to enhance the monitoring process in dynamic complex social networks. For this purpose, we utilize the Organizational Cybernetic Approach (OCA) [4] to control the communications between communities in both online and offline social network, and the stochastic one-median problem [5] to minimize the response time of the operation level to any malicious information spread across the network.

The rest of the paper is organized as follows. Literature review in section 2, section 3 explains the solution strategy, section 4 discusses the results, and section 5 presents conclusion and future work.

2 Literature Review

Due to the complexities in the social network analysis [4], this research will implement the systems thinking approaches for minimizing the response time to malicious information spread on online (offline) social networks. Alassad et al. [3] presented a theoretical approach to systems thinking and modeling in social networks to control conspiracy theories on Twitter. Authors in this study illustrated the noncooperative actions between two online organizations to influence both online and offline environments. Likewise, other empirical studies have introduced many systems thinking and modeling methods as advanced classes of intelligence to solve many complex problems in social networks as explained in [6]. Casual feedback method presented in [7] studied some of the variables needed in complex systems analysis. Muchnik et al. [8] focused on implementing the power laws methods in social networks and large scale communities. Du et al. [9] explained the boundary conditions methods. Also, well-known methods such as the control theory [10], and information theory [11], are essential systems thinking methods in social networks analysis as explained in [3]. The decision and game theory presented in [12] could bring optimized solutions and enable modeling different agent base complex problems. However, due to different behaviors of the users and the dynamic changes of the communities in social networks over time, the multidisciplinary methods are needed to incorporate comprehensive methods to analyze the local relationships, communications, and all positive/negative feedbacks (to and) from the environments.

Moreover, many operational methods were applied to model the interactions between the operations and the environments' dynamicity in different areas. Hakimi [13] proposed the ρ-median location model to find the optimal communities of the operation level when a certain set of failures occur in the infrastructure networks. The author minimized the shortest path between nodes by measuring the number of hubs. Patterson et al. [14] proposed a relaxed ρ-median model by allowing different service regions to overlap and reducing the loss of calls in a telecommunication network. Love et al. [15] proposed a model to illustrate the interactions between operation levels, where the ρ-median algorithm represented as bipartite graph. On the other hand, Odoni [16] stated that the ρ-median model can build a queuing situation affected by long arcs operations. However, Chan et al. [17] suggested that this model becomes analytically complex when the number of operation centers goes beyond one center to locate. Ahituv et al. [5] proposed a model to partition the network down into smaller subnetworks, each capable of independent operation level.

3 Proposed Solution

The motivation behind this research is to introduce an advanced Systems Thinking modeling into the social network analysis. A systems design modeling able to visualize or conceptualize any actions in the social network environment and perform a situational assessment of a social network conditions. The major challenge in this multidisciplinary modeling is to formulate, develop, and synthesize a set of solutions able to respond to user operational needs and constraints at the same time. Similarity, this systems design should perform and evaluate and select the optimal solutions that provide acceptable risk to satisfy the social network stakeholders' operational needs, where these demands introducing advanced systematic modeling methods in the social network analysis. The systems design modeling should able to optimize the selected solutions, provide the best values, enhance the agent performance, observe the performance or the lack thereof, assimilate the observable facts, and analyze the contributory causes and effects in social network [18]. This section presents the Systems Thinking methods to model the relationships between all parts and optimize the solutions without wasting time and efforts on trial-and-error experiments.

3.1 Assumptions

Here we list the set of deterministic assumptions that are required for the solution procedure, where these factors form the operation level strategies:

- The operation level response time to any malicious information spread is deterministic, ($V = 55$) messages/hour.
- The maximum time required to response to any malicious information spread from any community is normally distributed ($\beta = 2$) case/hours.
- The time of the operation level to prepare for the second and new stochastic malicious information spread is deterministic.
- Communities are assumed to be active and do not disappear from the network over time.
- Communities are assumed to be either spreading malicious information or not.

3.2 Problem Definition

Consider a social network $G = (N, A)$ consisted of the distinct nodes sets, $N = \{1,2, \dots, n\}$ and the set of edge (links) A, $A = \{(i,j), (k,l), \dots, (s,t)\}$ represented by directed node pair combinations going from community i to community j. Also, communities i, j are associate with numerical values representing number of intra edges $(d_{j,i})$ between communities i, and j. (h_j) represents the communities' maximum rate of the malicious information spread $\left(h_j = \frac{|N_j|}{N}\right)$, and (λ_i) is the proportion rate of operation level that can be monitored in the network.

Given all above, goal is to enhance the performance of the operation level, provide the implementation level of service in monitoring the (offline) online social networks, and optimize the community selections to minimize the response time to any malicious information spread from community i to community j and vise versa in G.

For this purpose, after clustering the network into different communities using the Modularity Method [19], we implemented the Organizational Cybernetic Approach (OCA) method and one-median problem in our solution procedure as explained below.

3.3 Organizational Cybernetic Approach (OCA)

Organizational cybernetics is the method of control and communication between a system and environments that includes different negative and positive feedbacks, flags unexpected behaviors from all users and parts in a system, and simplifies the analysis of the increased dynamicity in the entire system [4]. Likewise, the main objective in OCA is to help managers to control through monitoring the outputs and manipulating their inputs appropriately rather than breaking the systems down into parts to understand the systems parts' behaviors [4] as presented in Figure 1.

Operation Level System. This system is considered as system 1 in OCA. It needs to be as free as possible to allow its elements to interact with their environments efficiently [4]. The operation level has to maintain equity in monitoring big communities across the network considering factors such as reporting any abnormal behaviors or posts in the network [17]. The operation level must always report to management and control levels after completing a response to any malicious information post or before starting a new action. Also, this level needs to continuously monitor processes to respond to any abnormal information spread. In addition, the monitoring process and the arrangements with each malicious information spread should be in a first-in-first-out (FIFO) order.

Moreover, the monitoring process achieved in this level can be broken down into different operation elements and different shifts of working hours based on fixed time frames. This is necessary where any element in the operation level may interact with the network in different ways such as sharing information between different levels,or blocking malicious users' accounts from the networks in real time.

3.4 Stochastic One-Median Problem

This method helps with stochastic information spreads in dynamic networks where the operation and the response to the malicious information may be undetected. In addition, its helpful when customers use the network more than physically interacting with the offline environment or claiming abnormal behaviors in online social networks.

The stochastic one-median is like an allocation problem rather than a location problem, where it depends on the spread and the response process. There are four outlines in the operation level and monitoring process that can be translated into the OCA structure. In other words, the operation level is the sole system level responsible for responding to abnormal behaviors in the network. Also, this level would monitor the interactions of the users/communities on the online/offline environments. In addition, this level will report back any changes in the behaviors of the networks to higher levels in the OCA. Likewise, the Off-scene setup time to respond to any stochastic and new abnormal behaviors in the network.

In this research, we utilized the stochastic one-median problem to maximize the efforts of the operation levels and help to select the best communities combinations for the best monitoring process. The stochastic monitoring process can be implemented from the following model where the expected response time to any abnormal behaviors is measured in equation (1).

$$Min\, TR_j(C) \qquad \forall j \in I \qquad (1)$$

$$TR(C_j) = \bar{Q}_{C_j} + \bar{t}_{C_j} \qquad \forall j \in I \qquad (2)$$

$$\bar{Q}_{C_j} = \frac{\lambda_i\, \bar{S_2}(C_j)}{2\left(1 - \lambda_i \bar{S}(C_j)\right)} \qquad \begin{array}{c}\forall j \in I \\ \forall i \in M\end{array} \qquad (3)$$

$$\bar{t}_{C_j} = \sum_{j=1}^{I} h_j d(C_j, I) \qquad \forall j \in I \qquad (4)$$

TR is the sum of the mean- queuing-delay $\bar{Q}$ and the mean response time $\underline{t}$ as shown in equation (2). Equation (3) is to define $\bar{Q}$, where C_j is the community j in the network, λ_i is the rate of the proportion monitoring process, $\bar{S}(C_j)$ is the mean total response time (starting from the first moment the abnormal behavior is detected), $\bar{S_2}(C_j)$ is the second stochastic moment of the total response time to any new abnormal spread. Equation (4) is to define $\bar{t}$, where I is the number of communities in the network, h_j represents the maximum rate of the malicious information spread of the communities j, and $d(C_j, I)$ is the shortest path between community C_j, and community i.

4 Results & Findings

For evaluating our methodology, we employed a Twitter data set into the model to optimize the monitoring process and minimize response time to any information disseminated. In addition, the policy level in OCA assumed to hire only two monitoring (operation) centers in the network ($M = 2$), where each center will monitor less than (λ_i< 55%) of the entire network.

4.1 Dataset

A Python script was used to collect a co-hashtag network using #MichiganProtest, #MiLeg, #Endthelockdown, and #LetMiPeopleGo hashtags over the time period of April 01, 2020 to May 20, 2020. The data collected resulted in a network of 16,383 Tweets, with 9,985 unique User Ids. However, we filtered the dataset to focus only on events from May 12th to 15th. The resultant graph, as shown in Figure 2, revealed 3,632 nodes with 382 communities using the Modularity method [19]. However, for this analysis we used only a time window from May 12th to May 15th, 2020 and chose to focus on top 5 communities, as these were the communities with the highest number of users. Also, the model will represent all other small communities in one node to include any actions from (to) these communities into the solution procedure.

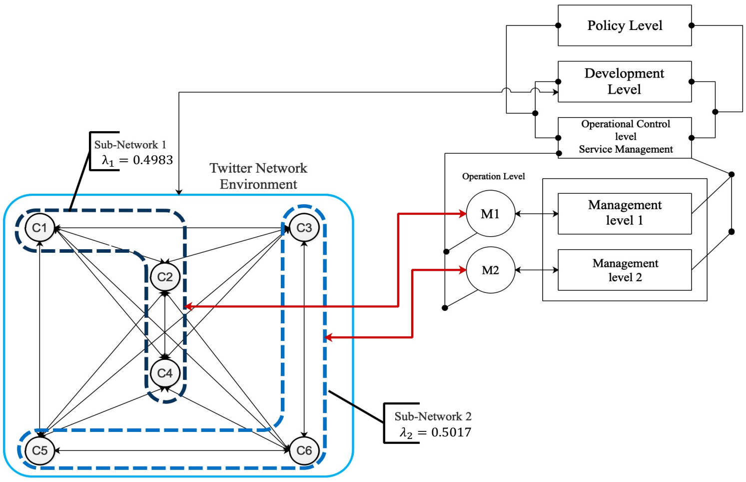

Figure 1: Organizational Cybernetic Approach.

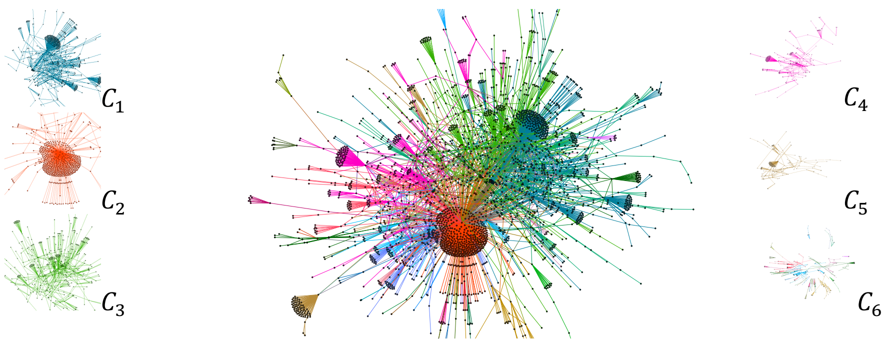

Figure 2: Twitter network for the COVID-19 anti-lockdown protest at Michigan state.

4.2 Experimental Results

The results in this section were based on multiple steps explained as follows:
Step 1: Clustering the network into smaller communities based on Modularity method [23] as presented in Figure 2.
Step 2: Measuring the intra edges between all 6 communities presented in Figure 2, representing the $d(C_j, I)$ values as shown in Table 1.

Table 1: Intra edges between the communities presented in Figure 2.

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	-	87	114	11	12	39
C_2	87	-	61	43	20	191
C_3	114	51	-	27	13	43
C_4	11	43	27	-	6	34
C_5	39	20	13	6	-	26
C_6	39	191	43	34	26	-

Step 3: Measuring the maximum malicious information spread (if all users in the community were spreading malicious action) proportion rate (h_j), for all six communities $\sum_{j=1}^{I} h_j = 1$ as shown in Table 2.

Table 2: Community C_j proportion rate h_j in spreading information in the network.

h_{C_1}	h_{C_2}	h_{C_3}	h_{C_4}	h_{C_5}	h_{C_6}
0.2189	0.1944	0.1621	0.085	0.0656	0.274

Step 4: Since we assumed ($M = 2$), this step divides the network into two sub-networks based on the λ_i values shown in Figure 1. Monitoring Center 1 (M_1) is for sub-network 1 including (C_1, C_2, C_4), and the Monitoring Center 2 (M_1) is for sub-network 2 including (C_3, C_5, C_6). Each operation level (monitoring center) will cover less than 55% of the entire network as maximum spreading rate. In case the policy level decided to increase the operation levels to ($M = 3$ or 4), then divide the network into 3 or 4 sub-networks respectively.

Table 3: Possible solutions combinations.

$C_{2,j}$	MTR (hours)	$C_{1,j}$	MTR (hours)	$C_{4,j}$	MTR (hours)
C_2 & C_3	7.699 & 1.944	C_1 & C_3	∞ & 1.944	C_4 & C_3	∞ & 1.944
C_2 & C_5	7.699, & 1.399	C_1 & C_5	∞ & 1.399	C_4 & C_5	∞ & 1.399
C_2 & C_6	7.699 & 0.618	C_1 & C_6	∞ & 0.618	C_4 & C_6	∞ & 0.618

Step 5: Apply the stochastic one-median model presented in section 3.4 model. The outcomes will provide the possible solutions to minimize the response time from each monitoring center. Table 3 presents 9 possible solutions, where the best communities' combinations allocation to minimize the response time would be at C_2 and C_6. These two communities are responsible for large amounts of information flow from (to) other communities in the network. However, the communities that received the negative

response time (∞) would be ignored from the solution procedure. The reason the model didn't select these locations is due to their scales, where in case the policy level allocates two monitoring centers within these communities, they cannot perform any response to any malicious information spread at other locations due to the heavy number of users and communities' activities. All the calculations are provided in Appendix A.

5 Conclusion

Systems thinking methods are useful tools to merge different disciplines' methods into the social network analysis. In this research, we modeled the operation level as system 1 introduced in organizational cybernetic approach (OCA) into the social media analysis. Also, we implemented the stochastic one-median problem to minimize the response time to any malicious information spread in social networks. In addition, this research presented an allocation solution procedure to optimize the resources of any organization to fight against malicious information spread, improve the monitoring process over time, and increase the performance of the operation level.

For future work, we will study the intra edges' reliability and measure the increase/decrease of information spread in dynamic networks. This will allow us to use different delay time assumptions and build an efficient frontier to illustrate the performances of the operation level. Also, we plan to model a stochastic social network analysis where communities in both online and offline environments are experiencing multi malicious information spread between the communities at the same time.

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References

[1] "Robinhood, Reddit CEOs to Testify Before Congress on GameStop." [Online]. Available: https://www.investopedia.com/robinhood-reddit-ceos-to-testify-in-congress-on-gamestop-gme-5112714. [Accessed: 16-Feb-2021].

[2] M. Alassad, B. Spann, and N. Agarwal, "Combining advanced computational social science and graph theoretic techniques to reveal adversarial information operations," *Inf. Process. Manag.*, p. 102385, Sep. 2020.

[3] M. Alassad, M. N. Hussain, and N. Agarwal, "How to Control Coronavirus Conspiracy

Theories in Twitter? A Systems Thinking and Social Networks Modeling Approach," in *2020 IEEE International Conference on Big Data (Big Data)*, 2020, pp. 4293–4299.
[4] C. J. H. Mann, *Systems Thinking – Creative Holism for Managers*, vol. 33, no. 8. 2004.
[5] N. Ahituv and O. Berman, *Operations Management of Distributed Service Networks*. 1988.
[6] P. A. Losty and G. M. Weinberg, "An Introduction to General Systems Thinking.," *J. R. Stat. Soc. Ser. A*, vol. 139, no. 4, p. 544, 1976.
[7] T. J. VanderWeele and W. An, "Social Networks and Causal Inference," Springer, Dordrecht, 2013, pp. 353–374.
[8] L. Muchnik *et al.*, "Origins of power-law degree distribution in the heterogeneity of human activity in social networks," *Sci. Rep.*, vol. 3, no. 1, pp. 1–8, May 2013.
[9] B. Du, X. Lian, and X. Cheng, "Partial differential equation modeling with Dirichlet boundary conditions on social networks," *Bound. Value Probl.*, vol. 2018, no. 1, pp. 1–11, Dec. 2018.
[10] L. Weng *et al.*, "The role of information diffusion in the evolution of social networks," in *Proceedings of the ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, 2013, vol. Part F128815, pp. 356–364.
[11] S. Peng, A. Yang, L. Cao, S. Yu, and D. Xie, "Social influence modeling using information theory in mobile social networks," *Inf. Sci. (Ny).*, vol. 379, pp. 146–159, Feb. 2017.
[12] Y. Chan and J. McCarthy, "Game-Theoretic Paradigms in Collaborative Research: Part 2-experimental design," *Int. J. Soc. Syst. Sci.*, vol. 6, no. 4, pp. 348–364, 2014.
[13] S. L. Hakimi, "Optimum Locations of Switching Centers and the Absolute Centers and Medians of a Graph," *Oper. Res.*, vol. 12, no. 3, pp. 450–459, Jun. 1964.
[14] T. S. Patterson, "Dynamic Maintenance Scheduling for a Stochastic Telecommunication Network: Determination of Performance Factors.," Air Force Institute of Technology Air University, 1995.
[15] R. Love, J. Morris, and G. Wesolowsky, "Facilities location," 1988.
[16] "Facility-Location Models," in *Location, Transport and Land-Use*, Springer-Verlag, 2005, pp. 21–119.
[17] "Measuring Spatial Separation: Distance, Time, Routing, and Accessibility," in *Location, Transport and Land-Use*, Springer-Verlag, 2005, pp. 120–209.
[18] C. Wasson, *System analysis, design, and development: Concepts, principles, and practices*. 2005.
[19] M. Girvan and M. Newman, "Community structure in social and biological networks," *Pnas*, vol. 99, no. 12, pp. 7821–7826, 2002.

Appendix A

This step associated with two assumptions where (V=55, and β =2)
Sub-Network 1: includes communities C_1, C_2, C_4.

Community # 1 (C_1)

$$\bar{t}_{c_1} = \frac{h_{C_2}}{\lambda_1} \cdot \frac{d_{C_2,C_1}}{V} + \frac{h_{C_4}}{\lambda_1} \cdot \frac{d_{C_2,C_1} + d_{C_2,C_4}}{V} \qquad \bar{t}_{c_1} = 1.020$$

$$\bar{S}_{c_1} = \beta \cdot \bar{t}_{c_1} \qquad \bar{S}_{c_1} = 2.041$$

$$\bar{S}_{2_{c_1}} = \frac{h_{C_2}}{\lambda_1} \cdot \left[\frac{\beta \cdot (d_{C_2,C_1})}{V}\right]^2 + \frac{h_{C_4}}{\lambda_1} \cdot \left[\frac{\beta \cdot (d_{C_2,C_1} + d_{C_2,C_4})}{V}\right]^2 \qquad \bar{S}_{2_{c_1}} = 7.717$$

$$TR(C_1) = \frac{\lambda_1 \bar{S}_{2_{c_1}}}{2(1 - \lambda_1 \bar{S}_{c_1})} + \bar{t}_{c_1} \qquad TR(C_1) = -113.222$$

Community # 2 (C_2)

$$\bar{t}_{c_2} = \frac{h_{C_1}}{\lambda_1}\cdot\frac{d_{C_2,C_1}}{V} + \frac{h_{C_4}}{\lambda_1}\cdot\frac{d_{C_2,C_4}}{V} \qquad \bar{t}_{c_2} = 0.828$$

$$\bar{S}_{c_2} = \beta\cdot\bar{t}_{c_2} \qquad \bar{S}_{c_2} = 1.656$$

$$\bar{S}_{2_{c_2}} = \frac{h_{C_1}}{\lambda_1}\cdot\left[\frac{\beta\cdot(d_{C_2,C_1})}{V}\right]^2 + \frac{h_{C_4}}{\lambda_1}\cdot\left[\frac{\beta\cdot(d_{C_2,C_4})}{V}\right]^2 \qquad \bar{S}_{2_{c_2}} = 4.814$$

$$TR(C_2) = \frac{\lambda_1\,\bar{S}_{2_{c_2}}}{2(1-\lambda_1\bar{S}_{c_2})} + \bar{t}_{c_2} \qquad TR(C_2) = 7.699$$

Community # 4 (C_4)

$$\bar{t}_{c_4} = \frac{h_{C_2}}{\lambda_1}\cdot\frac{d_{C_2,C_4}}{V} + \frac{h_{C_1}}{\lambda_1}\cdot\frac{d_{C_2,C_4}+d_{C_2,C_1}}{V} \qquad \bar{t}_{c_4} = 1.343$$

$$\bar{S}_{c_4} = \beta\cdot\bar{t}_{c_4} \qquad \bar{S}_{c_4} = 2.041$$

$$\bar{S}_{2_{c_4}} = \frac{h_{C_2}}{\lambda_1}\cdot\left[\frac{\beta\cdot(d_{C_2,C_4})}{V}\right]^2 + \frac{h_{C_1}}{\lambda_1}\cdot\left[\frac{\beta\cdot(d_{C_2,C_4}+d_{C_2,C_1})}{V}\right]^2 \qquad \bar{S}_{2_{c_4}} = 7.717$$

$$TR(C_4) = \frac{\lambda_1\,\bar{S}_{2_{c_4}}}{2(1-\lambda_1\bar{S}_{c_4})} + \bar{t}_{c_4} \qquad TR(C_4) = -6.578$$

Sub-Network 2: includes communities C_3, C_5, C_6.

Community # 3 (C_3)

$$\bar{t}_{c_3} = \frac{h_{C_6}}{\lambda_2}\cdot\frac{d_{C_6,C_3}}{V} + \frac{h_{C_5}}{\lambda_2}\cdot\frac{d_{C_6,C_3}+d_{C_6,C_5}}{V} \qquad \bar{t}_{c_3} = 0.553$$

$$\bar{S}_{c_3} = \beta\cdot\bar{t}_{c_3} \qquad \bar{S}_{c_3} = 1.107$$

$$\bar{S}_{2_{c_3}} = \frac{h_{C_6}}{\lambda_2}\cdot\left[\frac{\beta\cdot(d_{C_6,C_3})}{V}\right]^2 + \frac{h_{C_5}}{\lambda_2}\cdot\left[\frac{\beta\cdot(d_{C_6,C_3}+d_{C_6,C_5})}{V}\right]^2 \qquad \bar{S}_{2_{c_3}} = 1.963$$

$$TR(C_3) = \frac{\lambda_2\,\bar{S}_{2_{c_3}}}{2(1-\lambda_2\bar{S}_{c_3})} + \bar{t}_{c_3} \qquad TR(C_3) = 1.944$$

Community # 6 (C_6)

$$\bar{t}_{c_6} = \frac{h_{C_3}}{\lambda_2}\cdot\frac{d_{C_6,C_3}}{V} + \frac{h_{C_5}}{\lambda_2}\cdot\frac{d_{C_6,C_5}}{V} \qquad \bar{t}_{c_6} = 0.286$$

$$\bar{S}_{c_6} = \beta\cdot\bar{t}_{c_6} \qquad \bar{S}_{c_6} = 0.572$$

$$\bar{S}_{2_{c_6}} = \frac{h_{C_3}}{\lambda_2}\cdot\left[\frac{\beta\cdot(d_{C_2,C_1})}{V}\right]^2 + \frac{h_{C_5}}{\lambda_2}\cdot\left[\frac{\beta\cdot(d_{C_6,C_5})}{V}\right]^2 \qquad \bar{S}_{2_{c_6}} = 0.825$$

$$TR(C_6) = \frac{\lambda_2\,\bar{S}_{2_{c_6}}}{2(1-\lambda_2\bar{S}_{c_6})} + \bar{t}_{c_6} \qquad TR(C_6) = 0.618$$

Community # 5 (C_5)

$$\bar{t}_{c_5} = \frac{h_{C_5}}{\lambda_2}\cdot\frac{d_{C_6,C_5}}{V} + \frac{h_{C_3}}{\lambda_2}\cdot\frac{d_{C_6,C_5}+d_{C_6,C_3}}{V} \qquad \bar{t}_{c_5} = 0.425$$

$$\bar{S}_{c_5} = \beta\cdot\bar{t}_{c_5} \qquad \bar{S}_{c_5} = 0.850$$

$$\bar{S}_{2_{c_5}} = \frac{h_{C_5}}{\lambda_2}\cdot\left[\frac{\beta\cdot(d_{C_6,C_5})}{V}\right]^2 + \frac{h_{C_3}}{\lambda_2}\cdot\left[\frac{\beta\cdot(d_{C_6,C_5}+d_{C_6,C_3})}{V}\right]^2 \qquad \bar{S}_{2_{c_5}} = 1.876$$

$$TR(C_5) = \frac{\lambda_2\,\bar{S}_{2_{c_5}}}{2(1-\lambda_5\bar{S}_{c_5})} + \bar{t}_{c_5} \qquad TR(C_5) = 1.399$$